

On a stochastic model of damage and rupture

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In this talk I present you a joint work on the analysis of a time noise-driven Allen-Cahn equation modelling the evolution of damage in continuum media in the presence of stochastic dynamics [1]. In detail, we are interested in the following stochastic PDE

$$\begin{cases} du + (\xi - \Delta u) dt & = & (w_s(u) + f) dt + h(u)dW & \text{in } \Omega \times D \times (0, T) \\ u(\omega, x, t = 0) & = & u_0(x) & \omega \in \Omega, x \in D, \\ \nabla u \cdot \mathbf{n} & = & 0 & \text{in } \Omega \times \partial D \times (0, T), \end{cases} \quad (1)$$

where $\xi \in \partial I_{[0,1]}(u)$, $T > 0$, $W = \{W_t, \mathcal{F}_t, 0 \leq t \leq T\}$ is a standard adapted continuous Brownian motion defined on the classical Wiener space $(\Omega, \mathcal{F}, \mathbb{P})$, D is a smooth bounded domain of \mathbb{R}^d with $d \geq 1$, \mathbf{n} is the outward unit normal vector to ∂D and u_0 is a given initial condition. Note that the equation in (1) can also be written in the following way:

$$w_s(u) + f - \partial_t \left(u - \int_0^t h(u) dW \right) + \Delta u \in \partial I_{[0,1]}(u) \text{ in } \Omega \times D \times (0, T),$$

where the stochastic integral is understood in the sense of Itô (see DA PRATO-ZABCZYK [2]). The subdifferential $\partial I_{[0,1]}$ represents a physical constraint on u which is forced to take values in the interval $[0, 1]$. Using a Yosida approximation and time-discretization procedure, I will present a result of global-in-time existence and uniqueness of the solution under the following hypotheses:

H_1 : $u_0 \in H^1(D)$.

H_2 : $0 \leq u_0(x) \leq 1$ for almost all $x \in D$.

H_3 : $h : \mathbb{R} \rightarrow \mathbb{R}$ is a Lipschitz-continuous function such that $h(0) = h(1) = 0$.

H_4 : $w_s : \mathbb{R} \rightarrow [0, +\infty)$ is a Lipschitz-continuous function with, for convenience, $w_s(0) = 0$.

H_5 : f is a predictable process belonging to $L^2((0, T) \times \Omega, L^2(D))$.

Keywords : Subdifferential, Yosida approximation, maximal monotone operators, stochastic perturbation, Itô integral, multiplicative noise, predictable processes, implicit time discretization...

References

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