Sixièmes Journées Franco-Chiliennes d'Optimisation

19 - 21 mai 2008

Laboratoire Imath - Université du Sud Toulon-Var

Complexe Agélonde - La Londe les Maures

organisées par

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soutenues par

CMM Centro de Modelamiento Matemático

Imath

Université du Sud Toulon-Var

Région Provence-Alpes-Côte d'Azur

CNRS Centre National de la Recherche Scientifique

Toulon Provence Méditerranée Communauté d'Agglomération

Var Conseil Général

Présentation

Les Sixièmes Journées Franco-Chiliennes d'Optimisation ont été organisées par l'Université du Sud Toulon-Var, dans les locaux d'Agélonde, à La Londe les Maures, du lundi 19 au mercredi 21 mai 2008. Elles ont réuni 51 participants dont 33 conférenciers, jeunes et confirmés, parmis lesquels 12 chercheurs chiliens, deux chercheurs italiens et un chercheur grec.

Cette manifestation a été financée conjointement par l'Institut de Mathématiques de Toulon et du Var (Imath, USTV), le Centro de Modelamiento Matematico (UMI 2807, Universidad de Chile), l'Université du Sud Toulon-Var, le Centre National de la Recherche Scientifique, la communauté d'agglomération Toulon Provence Méditerranée, la région Provence-Alpes-Côte d'Azur et le Conseil Général du Var.

Les Journées Franco-Chiliennes d'Optimisation rentrent dans le cadre de la coopération franco-chilienne et sont destinées à resserrer les liens entre les équipes françaises et chiliennes travaillant dans le domaine de l'optimisation et les disciplines connexes. Cette coopération est en particulier financée par les programmes ECOS Sud (Evaluation-orientation de la Coopération Scientifique, programme français) et CONYCIT (Comisión Nacional de Investigación Científica y Tecnológica). Ces Journées sont organisées depuis 1995 par les diverses équipes françaises travaillant dans le domaine de l'optimisation et participant à des programmes de recherche et d'échange avec les équipes chiliennes: elles ont été organisées successivement à Limoges (1995), Paris (1997), Avignon (1998), Montpellier (2003) et Dijon (2006).

L'organisation de cette édition a été assurée par Guy Bouchitté et Thierry Champion du laboratoire Imath et Felipe Alvarez et Rafael Correa du Centro de Modelamiento Matematico. Le site internet dédié à cette manifestation se trouve à l'adresse suivante: `http://champion.univ-tln.fr/JFCO.html`

Le programme scientifique de cette édition a été principalement dédié à l'optimisation (tant sous ses aspects théoriques que numériques) ainsi qu'aux disciplines connexes telles que le calcul des variations (transport optimal de masse, problèmes variationnels provenant de la physique), le contrôle optimal (problèmes d'origine industrielle, approches théorique et numérique), l'économie mathématique (théorie des jeux, études de modèles de transport) et les problèmes inverses (applications à la tomographie). Les fichiers des conférences données sont accessibles sur le site internet indiqué ci-dessus.

Programme

Les exposés ont lieu à l'Astrolabe, dans la salle Vasco de Gama.

Lundi 19	**Mardi 20**	**Mercredi 21**
$8h20 - 9h$ *Accueil et ouverture*	$8h30 - 9h$ Auslender	
$9h00 - 9h30$ Cominetti	$9h00 - 9h30$ Carrasco	$8h45 - 9h15$ Seeger
$9h30 - 10h00$ Combettes	$9h30 - 10h00$ Jourani	$9h15 - 9h45$ Flores-Bazán
$10h00 - 10h30$ Carlier	$10h00 - 10h30$ Hadjisavvas	$9h45 - 10h15$ Mahadevan
pause café	*pause café*	*pause café*
$11h00 - 11h30$ Ramírez	$11h00 - 11h30$ Sorin	$10h45 - 11h15$ Quincampoix
$11h30 - 12h00$ Bolte	$11h30 - 12h00$ Jara	$11h15 - 11h45$ García
$12h00 - 12h30$ Ley	$12h00 - 12h30$ Maréchal	$11h45 - 12h15$ Attouch
Déjeuner	*Déjeuner*	*Déjeuner*
$14h00 - 14h30$ Correa	$14h00 - 14h30$ Alvarez	$13h45 - 14h15$ Chambolle
$14h30 - 15h00$ Bonnans	$14h30 - 15h00$ Seppecher	$14h15 - 14h45$ Buttazzo
$15h00 - 15h30$ Cardaliaguet	$15h00 - 15h30$ Peypouquet	
pause café	*pause café*	
$16h00 - 16h30$ De Pascale	$16h00 - 16h30$ Gajardo	
$16h30 - 17h00$ Jimenez	$16h30 - 17h00$ Piazza	
$17h00 - 17h30$ Hiriart-Urruty	$17h00 - 17h30$ Théra	
$19h15 - 20h00$ *Apéritif de bienvenue*	$17h30 - 17h40$ *Annonce Colloque "Variational Analysis"*	

Présentation des exposés

A user's guide to Riemannian Newton-type methods on manifolds

Felipe Alvarez and Julio Lopez

Several nonlinearly constrained optimization problems in the sciences and engineering can be formulated on a search space with a particular smooth geometric structure. Endowing such a manifold with special Riemannian metrics provides an intrinsic framework to develop numerical algorithms whose iterates automatically satisfy the constraints. In this talk, we will focus on Newton-type algorithms for optimization on some manifolds that appear in the applications, in particular matrix optimization problems. Special attention will be paid to the practical implementation of the discussed algorithms as to their differential geometric foundations. We will illustrate a variety of concrete algorithms through numerical tests running on specific simple examples inspired from real-world applications.

On the asymptotic behaviour of some multiscale dynamics. Application to weakly coupled systems

Hédy Attouch and Marc-Olivier Czarnecki

We present recent results obtained in collaboration with M.-O. Czarnecki concerning the asymptotic behaviour, as t goes to $+\infty$, of some non autonomous gradient dynamical systems involving multiscale features.
As a benchmark case, given H a general Hilbert space, $\Phi : H \to \mathbb{R} \cup \{+\infty\}$ and $\Psi : H \to \mathbb{R} \cup \{+\infty\}$ two closed convex functions, and β a function of t which tends to $+\infty$ as t goes to $+\infty$, we consider the following differential inclusion

$$\dot{z}(t) + \partial\Phi(z(t)) + \beta(t)\partial\Psi(z(t)) \ni 0.$$

This contains the case of nonautonomous "weakly coupled" dynamical systems

$$\left\{ \begin{array}{l} \dot{x}(t) + \partial f(x(t)) + \beta(t)A^t(Ax(t) - By(t)) \ni 0 \\ \dot{y}(t) + \partial g(y(t)) + \beta(t)B^t(By(t) - Ax(t)) \ni 0 \end{array} \right. \tag{1}$$

which correspond to $H = X \times Y$, $z = (x, y)$, $\Phi(z) = f(x) + g(y)$ and $\Psi(z) = \frac{1}{2}\|Ax - By\|^2$ with A and B two linear continuous operators.

This system allows to model the emergence of various coordination, synchronization and cooperation aspects. Its discretized versions bear a natural link with Passty's theorem and numerical splitting methods. We show several results ranging from ergodic convergence of the trajectories to convergence and rate of convergence results. As a key ingredient we assume that, for every $p \in N_C$

$$\int_0^{+\infty} \beta(t) \left[\Psi^*(\tfrac{p}{\beta(t)}) - \sigma_C(\tfrac{p}{\beta(t)}) \right] dt < +\infty$$

where Ψ^* is the Fenchel conjugate of Ψ, σ_C is the support function of $C = \operatorname{argmin}\Psi$ and N_C is the normal cone to C.

As a by-product, we revisit the sytem

$$\dot{z}(t) + \epsilon(t)\partial\Phi(z(t)) + \partial\Psi(z(t)) \ni 0$$

where $\epsilon(t)$ tends to zero as t goes to $+\infty$ and $\int_0^{+\infty} \epsilon(t)dt = +\infty$, whose asymptotic behaviour can be derived from the preceding study by some time rescaling.

Penalty and Smoothing Methods for Convex Semi-Infinite Programming

Alfred Auslender, Miguel Goberna and Marco Lopez

On considère des problèmes de Convex Semi-Infinite Programming. Pour les résoudre on propose deux types de méthodes: des algorithmes de type Remez couplés avec des méthodes de pénalité et de smoothing et des méthodes intégrales couplées elles aussi avec des méthodes de pénalité et de smoothing. Ces méthodes recouvrent en particulier des méthodes connues. La convergence des suites primales et duales est démontrée sous des conditions minimales.

Inégalités de Lojasiewicz : le point de vue de l'analyse

Jérôme Bolte, Aris Daniilidis, Olivier Ley et Laurent Mazet

L'inégalité de Lojasiewicz et ses diverses généralisations sont centrales dans l'analyse asymptotique des systèmes de type gradient (optimisation, complexité, equations aux dérivées partielles) et constituent une des motivations fortes de cette étude.

Le but de cette dernière est double. Il consiste d'une part à montrer que les inégalités Lojasiewicz sont valables dans de nombreux cas d'intérêt, et d'autre part à montrer qu'elles peuvent se caractériser en termes simples, familiers aux analystes : caractère lipschitz de l'application sous-niveaux, régularité métrique dans de bonnes métriques, courbes de gradient "par morceaux" de longueur uniformément bornées, talwegs de longueur finie. Si le temps le permet, nous évoquerons par ailleurs quelques applications.

Lors de son propre exposé, Olivier Ley examinera en détail le cas important des fonctions convexes.

Second-order optimality conditions for state-constrained optimal control problems

Joseph-Frédéric Bonnans and Audrey Hermant

We will discuss optimal control problem of an ordinary differential equation with several pure state constraints, of arbitrary orders, as well as mixed control-state constraints. We assume (i) the Hamiltonian to be strongly convex and the mixed constraints to be convex w.r.t. the control variable, and (ii) a linear independence condition of the active constraints at their respective order to hold. We give a complete analysis of the smoothness and junction conditions of the control and of the constraints multipliers. This allow us to obtain, when there are finitely many nontangential junction points, a theory of no-gap second-order optimality

conditions and a characterization of the well-posedness of the shooting algorithm. These results generalize those obtained in the case of a scalar-valued state constraint and a scalar-valued control.

Optimal Dirichlet regions for elliptic PDEs

Giuseppe Buttazzo

We consider an elliptic problem in a given domain Ω and a given right hand side f. The Dirichlet region is the unknown of the problem and has to be choosen in an optimal way, in order to minimize a cost functional, and in a class of admissible choices. The cost we consider is the compliance functional and the class of admissible choices consists of all one-dimensional connected sets (networks) of a given length L. Then we let L tend to infinity and look for the asymptotical distribution of the optimal networks. The asymptotically optimal shapes are discussed as well and links with average distance problems are provided.

Regularity of optimal control problems with super linear growth

Pierre Cardaliaguet

We investigate the regularity of the value function of some optimal control problems with super linear growth in the control variable. We show that the value functions are locally Hlder continuous with Hlder exponent depending only on the growth of the Hamiltonian. The proof relies on a reverse Hlder inequality.

Selection and approximation of Cheeger sets

Giuseppe Buttazzo, Guillaume Carlier, Myriam Comte and Gabriel Peyré

This talk is based on joint works with G.Buttazzo, M. Comte and G. Peyr. Given a bounded Lipschitz domain of $\mathbb{R}^2$, a Cheeger set of Ω is a minimizer of the ratio Perimeter over volume among finite perimeter subsets of the closure of Ω. We'll consider some variants of this problem where volume and perimeter are weighted by some given functions. After presenting several motivations for this problem, we'll discuss nonuniqueness issues, selection of a particular solution, the maximal Cheeger set and present a simple numerical approximation scheme.

Optimization of trusses with random loads

Felipe Álvarez and Miguel Carrasco

In this talk, we show that a problem of finding the truss of minimum expected compliance under stochastic loading conditions is equivalent to the dual of a special convex minimax problem, and therefore may be efficiently solved. This equivalence

makes it possible to provide classic multiload compliance minimization problems with interpretations in a probabilistic setting. In fact, we prove that minimizing the expected compliance amounts to solving a multiload like problem associated with a particular finite set of loading scenarios, which depends on the mean and the variance of the perturbations. Several numerical examples are presented.

The stochastic setting introduced in this talk allows to consider other variants for the optimal design problem. We then consider the minimum variance-compliance problem, where the variance of the compliance is included in the model. We show that this stochastic problem is equivalent to a mathematical programing problem. The variance formulation introduces a non convex term that makes this type of problems harder to solve numerically than the minimum expected compliance model. To solve numerically this optimization problem, we consider a hybrid optimization method based on a steepest-descent algorithm. We valid the variance-compliance model on a 3-D benchmark test case and compare obtained results with this given by the expected compliance model. We obtain a truss with a low risk level.

Rigidité dans les milieux fracturés

Antonin Chambolle

Un résultat classique dû à Liouville dit que si le gradient d'un déplacement est une rotation en tout point, alors ce déplacement est affine. Cette "rigidité" est une propriété fondamentale en mécanique des milieux continus. On essaiera de décrire sous quelles conditions ce type de propriété s'étend (ou non) à des matériaux "fracturés", dont le déplacement n'est par conséquent plus continu.

On the Moment Problem

Patrick Louis Combettes and Noli Reyes

Under consideration is the problem of finding a point in a Hilbert space which possesses prescribed best approximations from a finite number of closed vector subspaces. The main objective is to provide conditions for the existence of solutions to this problem in terms of the geometry of the subspaces, irrespective of the prescribed best approximations. The issues of uniqueness and stability are also briefly addressed, as well as numerical methods. The results are applied to problems in harmonic analysis, signal theory, and statistics.

Short-term revenue management: optimal targeting of customers for a last-minute offer

Roberto Cominetti, José Rafael Correa and Jaime San Martín

We study a short-term revenue optimization problem that involves the optimal targeting of customers for a promotional sale in which a finite number of perishable items are offered on a last-minute offer. The goal is to select the subset of customers to whom the offer will be made available, maximizing the expected return. Each

client replies with a certain probability and reports a specific value that depends on the customer type, so that the selected subset has to balance the risk of not selling all the items with the risk of assigning an item to a low value customer.

Selecting all those clients with values above a certain optimal threshold may fail to achieve the maximal revenue. However, using a linear programming relaxation, we prove that such threshold strategies attain a constant factor of the optimal value. The achieved factor is $\frac{1}{2}$ when a single item is to be sold, and approaches 1 as the number of available items grows to infinity. Furthermore, for the single item case, we propose an upper bound based on an exponential size linear program that allows us to get a threshold strategy achieving at least $\frac{2}{3}$ of the optimal revenue. Computational experiments with random instances show a significantly better performance than the theoretical predictions.

Characterization of Lipschitz like properties

Rafael Correa, Pedro Gajardo and Lionel Thibault

The aim of this talk is to show characterizations of various Lipschizian like properties of functions and sets, in terms of subdifferentials, directional derivatives, tangential and normal cones. We consider the study of K-directionally Lipschitzian property for functions and sets which recover some well known concepts in variational analysis as directionally Lipschitzian and compactly epi- Lipschitzian behaviors. We extend some necessary and sufficient conditions, given for the geometrical Ioffe subdifferential (resp. Frechet and Mordukhovich subdifferential) and Geometrical Ioffe normal cone (resp. Frechet and Mordukhovich normal cone), to other subdifferentials (and normal cones) that satisfy multidirectional mean value inequalities. Characterization in terms of generalized derivatives and tangent cones will be also studied.

Optimal transportation problems in some sand-piles model

Luigi De Pascale

We will show the connection betwen the optimal transportation problem (or Monge-Kantorovich problem) and a commonly used model in granular matter theory. We will use this connection to prove existence, uniqueness and a weak form of regularity for solutions in different situations.

Multivalued complementarity problems with asymptotically bounded multifunctions

Fabián Flores-Bazán

Given a multifunction $F : \mathbb{R}^n_+ \hookrightarrow \mathbb{R}^n$ and $q \in \mathbb{R}^n$, the multivalued complementarity problem (MCP) on the positive orthant consists in finding

$$\bar{x} \geq 0,\ \bar{y} \in F(\bar{x}) :\ \bar{y} + q \geq 0,\ \langle \bar{y} + q, \bar{x} \rangle = 0.$$

It is well documented that such a problem appears in many applications in Science and Engineering and therefore was the object of many investigations in the last three decades. Most of the works existing in the literature deal with the case when F is pseudomonotone (in the Karamardian sense) or quasimonotone, and only a few assume copositivity.
In this work we introduce the notion of asymptotic multifunction with respect to a class of re-scaling functions including those with slow growth, and the notion of asymptotic multifunction associated to a sequence of multifunctions rather to a single one. Based on these two concepts we establish new existence theorems for the MCP for a class of multifunctions larger than copositive without assuming positive (sub)homogeneity as in a previous work. In addition, some stability and sensitivity results, as well as a robustness property, are provided. Thus, in this regards, we unify and generalize some of the results previously established.

This research is supported in part by CONICYT through FONDECYT 107-0689, Chile, and by Center for Mathematical Modeling (CMM), Universidad de Chile.

Viable harvest of monotone bioeconomics models: preservation and production issues

Michel de Lara, Pedro Gajardo and Héctor Ramírez C.

The aim of this lecture is to show some applications of the viability theory, for discrete time dynamical systems, to an age structured abundance population model in fisheries management. We obtain necessary and sufficient conditions for levels of landings (catch, harvest) to be sustainable (independently of the current abundance) and, for a given vector of abundance (a state), we show how to compute the maximal sustainable catch starting from this state. All these tools could help to regulatory organisms in order to determinate and/or evaluate fishing quotas.

Properties of the variational sum of monotone operators

Yboon García

We study the Variational Sum of monotone operators, in particular its relationship with the Extended Sum of monotone operators. We establish new properties of the Variational Sum, we furthermore show that its graph contains that one of the former. This last result allows us to show that if the Extended Sum is maximal monotone then these two sums coincide.

Maximal monotonicity of bifunctions

Nicolas Hadjisavvas and H. Khatibzadeh

For each monotone bifunction F defined on a subset C of a Banach space, an associated monotone operator A^F can be defined. The bifunction F is called maximal monotone if A^F is maximal monotone. We provide simple verifiable conditions for a bifunction to be maximal monotone and show the relation to the existence

of solutions of an equilibrium problem. Also, we establish some properties of the domain of a maximal monotone bifunction. Finally, we define and study cyclically monotone bifunctions.

Objets convexes de largeur constante (en 2D) ou d'épaisseur constante (en 3D) : du neuf avec du vieux.

Jean-Baptiste Hiriart-Urruty

"When Minkowski's theory of convexity appeared, some mathematicians said that he discovered a nice mathematical joy which, unfortunately, is quite useless. About a century passed, and now the theory of convex sets is a very important applied branch of mathematics."

V. Boltyanski, in **Geometric methods and optimization problems** (1999).

Résumé. La convexité "géométrique" (celle traitant des structures des corps convexes, de leur lissité, volume...) et la convexité "fonctionnelle" (celle traitant des propriétés des fonctions convexes, de leur utilisation en optimisation,...) sont deux domaines bien répertoriés mais différents des mathématiques, avec chacun leurs communautés, préoccupations, revues, etc. Il est néanmoins un point d'intérêt qui permet de jeter un pont entre les deux, c'est celui des problèmes variationnels posés à propos des corps convexes. Dans cet exposé, essentiellement d'ordre pédagogique et de synthèse, nous examinons les propriétés, plutôt de "type variationnel", des corps convexes de largeur constante (en 2D) ou d'épaisseur constante (en 3D), en insistant sur les différences fondamentales en 2D et 3D. Nous arrivons ainsi sur le front des problèmes ouverts (souvent depuis longtemps), sur lesquels les résultats et techniques du Calcul variationnel, Commande optimale et/ou Optimisation des formes ont échoué jusqu'à présent. Les avancées les plus décisives à ce sujet, dans un contexte 3D, ont été apportées par T. Bayen, Th. Lachand-Robert, E. Oudet (travaux publiés en 2007).

"La convexité dans le plan et dans l'espace *présente un sujet passionnant, la convexité, à la fois par sa simplicité, sa naturalité et sa puissance, pour au moins trois raisons :*

- au niveau des questions que l'on peut se poser naturellement, géométriques, visibles ;

- du fait que la convexité est une notion qui se rencontre dans de nombreuses branches des mathématiques ;

- du fait de son utilité, de sa force, dans de nombreuses applications."

M. Berger in **Convexité dans le plan et au-delà**, 2 volumes, collection Opuscules, éditions Ellipses (2006).

Rationalizability in games with a continuum of players

Pedro Jara

The concept of Rationalizability has been used in the last fifteen years to study stability of equilibria on models with a continuum of agents such as competitive markets, macroeconomic dynamics and currency attacks. However, Rationalizability has been formally defined in a general setting only for games with a finite number of players. We propose then a definition for Point-Rationalizable Strategies in the context of Games with a Continuum of Players. In a special class of these games, where the payoff of a player depends only on his own strategy and an aggregate value that represents the state of the game, state that is obtained from the actions of all the players, we define the sets of Point-Rationalizable States and Rationalizable States. These sets are characterized and some of their properties are explored. We study as well standard Rationalizability in a subclass of these games.

Optimum et équilibre pour un problème de transport optimal avec file d'attente

Gianluca Crippa, Chloé Jimenez et Aldo Pratelli

On considère k bureaux de postes dont les positions sont k points $x_1, ..., x_k$ d'un compact Ω de $\mathbb{R}^d$ représentant une ville. Soit $f dx$ la densité de population dans cette ville. On cherche une partition $(A_i)_i$ de Ω telle que chaque habitant demeurant en $x \in A_i$ se rend en x_i. Le temps perdu par une personne qui fait ce choix est la somme des durées du trajet et de la file d'attente. La durée du trajet est représentée par $|x - x_i|^p$, i.e. la distance parcourue élevée à une puissance $p \geq 1$. Le temps d'attente dépend du nombre de personnes $\int_{A_i} f(x)\, dx$ qui se rendent en x_i et change selon le bureau de poste considéré (en fonction par exemple du nombre d'employés du bureau). Ainsi le coût total pour en $x \in A_i$ est $|x - x_i|^p + h_i \left(\int_{A_i} f(x)\, dx \right)$ où h_i est croissante. Le problème d'optimisation considéré est le suivant:

$$\inf \left\{ \sum_{i=1}^{k} \int_{A_i} |x - x_i|^p + h_i \left(\int_{A_i} f(x)\, dx \right)\ f(x)\, dx \right\}$$

où l'infimum est pris sur toute les partitions $(A_i)_i$ de Ω.

Dans un premier temps, nous avons montré que, sous certaines hypothèses, la partition optimale existe et est unique. Nous en avons donné une charactérisation.

Nous avons ensuite considéré le problème sous l'angle de la théorie des jeux. Chaque citadin se rendant en x_i est satisfait si ce choix minimise son propre coût en l'absence de changement de comportement des autres citadins, i.e. lorsque:

$$|x - x_i|^p + h_i \left(\int_{A_i} f(x)\, dx \right) = \min_j \left\{ |x - x_j|^p + h_j \left(\int_{A_j} f(x)\, dx \right) \right\}.$$

Une situation dans laquelle tous les habitants sont satisfaits s'appelle un équilibre. Dans le problème que nous considérons, sous certaines hypothèses, nous avons l'existence et l'unicité d'un tel équilibre. De plus l'équilibre est également un

optimum de Pareto. Nous étudions ensuite la dynamique du problème lorsque chaque citoyen prend sa décision en fonction de la connaissance des durées d'attente $h_i\left(\int_{A_i} f(x)\,dx\right)$ de la veille. Nous donnons les conditions de convergence vers l'équilibre lorsque les habitants suivent certaines stratégies.

Efficiency for continuous facility location problems with attraction and repulsion

Abderrahim Jourani, Christian Michelot and Mbaye Ndiaye

The talk deals with the problem of locating new facilities in presence of attracting and repulsive demand points in a continuous location space. When an arbitrary norm is used to measure distances and with closed convex constraints, we develop necessary conditions of efficiency. In the unconstrained case and if the norm derives from a scalar product, we completely characterize strict and weak efficiency and prove that the efficient set coincides with the strictly efficient set and/or coincides with the weakly efficient set. When the convex hulls of the attracting and repulsive demand points do not meet, we show that the three sets coincide with a closed convex set for which we give a complete geometrical description. We establish that the convex hulls of the attracting and repulsive demand points overlap iff the weakly efficient set is the whole space and a similar result holds for the efficient set when we replace the convex hulls by their relative interiors. We also provide a procedure which computes, in the plane and with a finite number of demand points, the efficient sets in polynomial time. Concerning constrained efficiency, we show that the process of pro jecting unconstrained weakly efficient points on the feasible set provides constrained weakly efficient points.

Kurdyka-Lojasiewicz inequality and subgradient trajectories: the convex case

Jérôme Bolte, Aris Daniilidis, Olivier Ley and Laurent Mazet

In his talk, Jérôme Bolte gave some characterizations of the Kurdyka-Lojasiewicz inequality by means of some properties of the gradient flow. Here, we make more precise this link for convex functions $f : H \to \mathbb{R}$, where H is a Hilbert space. Then, we answer in a negative way the following question: does a convex function always satisfy the Kurdyka-Lojasiewicz inequality? It is true if, for example, the convex function satisfies a growth condition from above around its minima. But it is false in general and we describe a counter-example of a smooth convex function $f : \mathbb{R}^2 \to \mathbb{R}$ which does not satisfy Kurdyka-Lojasiewicz inequality.

An extremal eigenvalue problem for a two-phase conductor

Carlos Conca, Rajesh Mahadevan and Leon Sanz

Since a long time it is well-known that problems of optimal design may not admit solutions if microstructural designs are excluded from consideration (see [Murat-Tartar 97]). The problem of minimizing the first eigenvalue of a two-phase conductor with the conducting phases to be distributed in a fixed proportion in a given domain has a classical solution, in one dimensional domains and also in a ball in any dimension(see [Kreĭn 55, Alvino-Lions-Travetti 89]). These existence results have been regarded so far as being exceptional owing to the presence of complete symmetry and so in the general case, S. Cox and R. Lipton limit their analysis to a study of the optimality conditions that an optimal microstructural design should satisfy [Cox-Lipton 96]. It is still not clear why the same problem in domains with partial symmetry should fail to have a solution which does not develop microstructure and respecting the symmetry of the domain. We hope to revive interest in this question by giving a new proof of the result in a ball using a symmetrization result from A. Alvino and G. Trombetti [Alvino-Trombetti 83].

Sur la régularisation par mollification

Pierre Maréchal

De nombreux problèmes inverses font intervenir, de manière plus ou moins explicite, des opérateurs de Fourier tronqués à un domaine borné. C'est par exemple le cas en synthèse d'ouverture, en IRM et en tomographie. Lorsqu'ils sont inversibles, ces opérateurs ont un inverse non continu, et la question fondamentale qui se pose alors est celle de leur régularisation.

Nous considérerons une approche variationnelle particulière de ce problème. Le paramètre de régularisation qui s'impose peut s'interpréter en termes de niveau de résolution de la solution correspondante, et l'on parle de régularisation par mollification.

Nous présenterons quelques résultats sur le comportement de la solution ainsi régularisée lorsque ce paramètre tend vers zéro. En particulier, des conditions relativement peu restrictives permettent d'assurer la convergence forte vers la solution du problème non régularisé. Ces résultats se distinguent de la plupart des techniques de régularisation en ce que le paramètre n'est pas une simple pondération entre les termes d'adéquation et de régularisation. Comme on peut s'y attendre, les techniques variationnelles utilisées sont alors radicalement différentes.

Nous évoquerons aussi quelques questions liées à l'échantillonnage, c'est-à-dire au cas où le domaine de troncature de Fourier est non seulement borné, mais aussi discret. Nous présenterons enfin quelques réflexions sur les possibilités d'extension de cette théorie au cas d'opérateurs compacts plus généraux, pour lesquels la régularisation par mollification paraît moins naturelle au premier abord. Ces extensions impliquent des opérateurs qui sont eux-mêmes définis comme minimiseurs d'une fonctionnelle.

Strong asymptotic convergence of evolution equations governed by maximal monotone operators with Tikhonov regularization

Juan Peypouquet

We consider the Tikhonov-like dynamics $-\dot{u}(t) \in A(u(t)) + \varepsilon(t)u(t)$ where A is a maximal monotone operator and the parameter function $\epsilon(t)$ tends to 0 for $t \to \infty$ with $\int_0^\infty \epsilon(t)dt = \infty$. When A is the subdifferential of a closed proper convex function f, we establish strong convergence of $u(t)$ towards the least-norm minimizer of f. In the general case we prove strong convergence towards the least-norm point in $A^{-1}(0)$ provided that the function $\epsilon(t)$ has bounded variation, and provide a counterexample when this property fails.

A characterization of the asymptotic behavior of optimally managed, multiple species forests

Adriana Piazza

In abstract terms our model concerns the optimal management of a resource which can be either traded in the market or allocated to different activities that, after a fixed delay, provide a benefit and liberate the resource for immediate reuse. We study in particular the optimal harvesting problem where the resource is the land surface that may be allocated among several forest species (having different maturity ages) and/or traded in the market.

We characterize the asymptotic behavior of the state of the mixed forest, proving the existence of *sustainable states* and we discuss the conditions under which any optimal trajectory converges in the long run towards one of these states or towards the set of optimal periodic cycles. Our results are valid both in the discounted and undiscounted framework.

On Derivative Criteria for Metric Regularity and Extensions to Hölder Context; Applications

Hélène Frankowska and Marc Quincampoix

In the paper, we provide new criteria for metric regularity of set-valued maps. These results are based on variations of set valued maps. This approach enables us to study and characterize the higher order metric regularity i.e. the extension of metric regularity to Hölder context. This could be also easily entended to conical metric regularity with respect to a cone with compact sole. We give several applications to continuous dependence of a parametrized set of constraints and computations of tangent cone.

On Second-Order Optimality Conditions for Conic Programming

Joseph-Frédéric Bonnans and Héctor Ramírez C.

In this talk we study second order optimality conditions for nonlinear conic programming problems, establishing links between nondegeneracy conditions and different second order optimality conditions. This extends known properties from classical nonlinear programming to a more general cone programming setting.

Then we discuss characterizations of strong regularity in terms of second order optimality conditions for second-order cone and semidefinite programming problems.

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Deterministic and random methods for computing volumetric moduli of convex cones

Daniel Gourion and Alberto Seeger

This work concerns the practical computation of the volumetric modulus, also called normalized volume, of a convex cone in a Euclidean space of dimension beyond three. Deterministic and random techniques are considered.

3D-2D analysis for the optimal elastic compliance problem

Guy Bouchitté, Ilaria Fragalà and Pierre Seppecher

A prescribed amount of linear elastic material has to be placed in a design region of very small height in order to maximize the resistance of the plate. We prove that, for the optimal shape and at the limit when the height tends to zero, flexion and extension are coupled through a Kirchhoff-Love motion. We give optimality conditions and find that the (rescaled) optimal shape has a disconnected section. The results differ fundamentally from the results obtained by optimizing the thickness of a plate under the constraint of a connected section.

Approachablility and differential games

S. Assoulamani, Marc Quincampoix and Sylvain Sorin

We develop the links between repeated games and differential games in the framework of approachability. A first result due to N. Vielle (M.O.R., 17, 1992) exhibits the connection between the asymptotic approach in repeated games (limit of finitely repeated or discounted games) and differential games of fixed duration to prove the weak-approachability property. The purpose of the current work is to exhibit a similar relation for the uniform approachability property (robustness of ε-optimal strategies in long games) in repeated games and qualitative differential

games. Starting from a repeated game G, we first construct an alternative deterministic repeated game G^* and a related differential game Γ. We then establish an alternative characterization of B-sets in G or G^* as discriminant domains in Γ. We show that a set is $*$-approachable in G^* iff it contains a B-set. We finally provide a map from winning strategies in the differential game to approachability strategies and thus recover Spinats characterization of approachability (M.O.R., 27, 2002).

Le Théorème de Lions et Stampacchia fête ses quarante ans

Michel Théra

Dans cet exposé nous montrerons que dans le cas original du Théorème de Lions et Stampacchia où l'opérateur est linéaire, la pseudomonotonie (au sens de Brezis) est une condition nécessaire et suffisante pour que l'inéquation variationnelle associée à cet opérateur admette une solution pour tout ensemble convexe borné d'un espace de Hilbert.

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