AN EXTREMAL EIGENVALUE PROBLEM FOR A TWO-PHASE CONDUCTOR

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Since a long time it is well-known that problems of optimal design may not admit solutions if microstructural designs are excluded from consideration (see [8]). The problem of minimizing the first eigenvalue of a two-phase conductor with the conducting phases to be distributed in a fixed proportion in a given domain has a classical solution, in one dimensional domains and also in a ball in any dimension(see [5, 1]). These existence results have been regarded so far as being exceptional owing to the presence of complete symmetry and so in the general case, S. Cox and R. Lipton limit their analysis to a study of the optimality conditions that an optimal microstructural design should satisfy [3]. It is still not clear why the same problem in domains with partial symmetry should fail to have a solution which does not develop microstructure and respecting the symmetry of the domain. We hope to revive interest in this question by giving a new proof of the result in a ball using a symmetrization result from A. Alvino and G. Trombetti [2].

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