ON THE ASYMPTOTIC BEHAVIOUR OF SOME MULTISCALE DYNAMICS.
APPLICATION TO WEAKLY COUPLED SYSTEMS

HÉDY ATTOUCH AND MARC-OLIVIER CZARNECKI

We present recent results obtained in collaboration with M.-O. Czarnecki concerning the asymptotic behaviour, as t goes to $+\infty$, of some non autonomous gradient dynamical systems involving multiscale features.
As a benchmark case, given H a general Hilbert space, $\Phi : H \to \mathbb{R} \cup \{+\infty\}$ and $\Psi : H \to \mathbb{R} \cup \{+\infty\}$ two closed convex functions, and β a function of t which tends to $+\infty$ as t goes to $+\infty$, we consider the following differential inclusion

$$\dot{z}(t) + \partial\Phi(z(t)) + \beta(t)\partial\Psi(z(t)) \ni 0.$$

This contains the case of nonautonomous "weakly coupled" dynamical systems

$$\left\{ \begin{array}{l} \dot{x}(t) + \partial f(x(t)) + \beta(t)A^t(Ax(t) - By(t)) \ni 0 \\ \\ \dot{y}(t) + \partial g(y(t)) + \beta(t)B^t(By(t) - Ax(t)) \ni 0 \end{array} \right. \tag{1}$$

which correspond to $H = X \times Y$, $z = (x, y)$, $\Phi(z) = f(x) + g(y)$ and $\Psi(z) = \frac{1}{2}\|Ax - By\|^2$ with A and B two linear continuous operators.

This system allows to model the emergence of various coordination, synchronization and cooperation aspects. Its discretized versions bear a natural link with Passty's theorem and numerical splitting methods. We show several results ranging from ergodic convergence of the trajectories to convergence and rate of convergence results. As a key ingredient we assume that, for every $p \in N_C$

$$\int_0^{+\infty} \beta(t) \left[\Psi^*(\tfrac{p}{\beta(t)}) - \sigma_C(\tfrac{p}{\beta(t)})\right] dt < +\infty$$

where Ψ^* is the Fenchel conjugate of Ψ, σ_C is the support function of $C = \text{argmin}\Psi$ and N_C is the normal cone to C.

As a by-product, we revisit the sytem

$$\dot{z}(t) + \epsilon(t)\partial\Phi(z(t)) + \partial\Psi(z(t)) \ni 0$$

where $\epsilon(t)$ tends to zero as t goes to $+\infty$ and $\int_0^{+\infty} \epsilon(t)dt = +\infty$, whose asymptotic behaviour can be derived from the preceding study by some time rescaling.

Institut de Mathématiques et de Modélisation de Montpellier,, UMR CNRS 5149, Université de Montpellier 2

E-mail address: attouch@math.univ-montp2.fr