Rationalizability in Games with a Continuum of Players

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Outline

1. Motivation
2. Games with a continuum of players (Rath, 1992)
   - Framework
   - Guesnerie (1992)
3. Rationalizable Strategies in games with a finite number of players
4. State Rationalizability
   - Point-Rationalizable States
   - Rationalizable States
   - Rationalizability in Guesnerie (1992)
5. Other Results
6. Summary

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  Context: games with a finite number of players.

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  Context: games with a finite number of players.
- Guesnerie (1992) defines *Strong Rationality* or *Eductive Stability*: *uniqueness of the* **rationalizable solution**
  Context: a specific economic setting, which featured a continuum of agents.

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- In each of these, intuitive and context-specific definitions for Rationalizability.

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- Find a suitable model of game with a continuum of players.
- Characterize **Rationalizable Outcomes** for these games.

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- The payoff functions $\mathbf{u}(i)(\,\cdot\,)$ depend on the other players' strategies through the integral of the strategy profile $\int s(i)\,\mathrm{di}$.
- There are functions $u(i,\,\cdot\,) : S \times \mathrm{co}\,\{S\} \to \mathbb{R}$ such that:

$$\mathbf{u}(i)(s(i)\,,\mathbf{s}) \equiv u\left(i, s(i)\,, \int s(i)\,\mathrm{di}\right)$$

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- S is compact. A strategy profile is a measurable function $\mathbf{s} : I \rightarrow S$. $\mathbf{s} \in S^I$.

Definition 1

A *Nash Equilibrium* is a strategy profile $\mathbf{s}^* \in S^I$ such that,

$$\text{for } \lambda\text{-a.e. } i \in I, \quad \mathbf{u}(i)\left(s^*(i)\,, \int \mathbf{s}^*\right) \geq \mathbf{u}(i)\left(y, \int \mathbf{s}^*\right) \quad \forall y \in S \tag{1}$$

We call $\mathcal{A} \equiv \mathrm{co}\,\{S\}$.
Optimal strategy correspondence $B(i, \cdot\,) : \mathcal{A} \rightrightarrows S$:

$$B(i,a) \quad := \quad \mathrm{argmax}_{y \in S}\,\{u(i,y,a)\}\,. \tag{2}$$

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$$\mathbb{B}(i,\mu) \quad := \quad \text{argmax}_{y \in S}\,\mathbb{E}_{\mu}\,[u(i,y,a)]\,. \tag{3}$$

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We denote $\Gamma(a) = \int_I B(i,a)\,\text{di}$. Equivalently, an *equilibrium* is a point $a^* \in \mathcal{A}$ such that:

$$a^* \in \Gamma(a^*) \equiv \int_I B(i,a^*)\,\text{di} \equiv \int_I \mathbb{B}(i,\delta_{a^*})\,\text{di} \tag{4}$$

- $\mathcal{U}_{S\times\mathcal{A}}$: the space of real valued continuous functions defined on $S\times\mathcal{A}$, endowed with the supremum norm.
- $\mathbf{u} : i \in I \to \mathbf{u}(i) \in \mathcal{U}_{S\times\mathcal{A}} \qquad \mathbf{u}(i) : S\times\mathcal{A} \to \mathbb{R}$.

HM : The mapping $\mathbf{u}$ is measurable.

Theorem 2 (Rath, 1992)

Every game $\mathbf{u}$ has a (pure strategy) Nash Equilibrium.

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- $B(i,p) \equiv \mathrm{Supply}(i)(p)$,
- $\mathbb{B}(i,\mu) \equiv \mathrm{Supply}(i)(\mathbb{E}_\mu\left[p\right])$

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Definition 3 (Bernheim, 1984)

s_i is a *Rationalizable Strategy* for player i if there exists some consistent system of beliefs for this player and some subjective product probability measure over the set of strategy profiles of the opponents, that gives zero probability to actions of the opponents of i that are ruled out by this system of beliefs and such that the strategy s_i maximizes expected payoff with respect to this probability measure.

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(i) is the result of the iterative and independent elimination of strategies that are not best-replies to any forecast considering all of the remaining strategy profiles
(ii) is the largest set that satisfies being a fixed point of the process of elimination of strategies.

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$$\rightsquigarrow \forall\, i \in I,\ s(i) \in B(i, X) \equiv \bigcup_{a \in X} B(i, a)$$

$$\rightsquigarrow \quad a = \int s(i)\,\mathrm{di} \in \int B(i, X)\,\mathrm{di}.$$

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Point-Rationalizable set, $\mathbb{P}_{\mathcal{A}}$, must satisfy:

$$\mathbb{P}_{\mathcal{A}} \subseteq \bigcap_{t=0}^{+\infty} \tilde{Pr}^t(\mathcal{A}) =: \mathbb{P}'_{\mathcal{A}}. \tag{5}$$

$$\mathbb{P}_{\mathcal{A}} \equiv \tilde{Pr}(\mathbb{P}_{\mathcal{A}})\,. \tag{6}$$

Definition 5

The set of *Point-Rationalizable States*, $\mathbb{P}_{\mathcal{A}}$, is the maximal subset $X \subseteq \mathcal{A}$ that satisfies the condition:

$$X \equiv \tilde{P}r(X)\,.$$

Theorem 6

Let us write

$$\mathbb{P}'_{\mathcal{A}} := \bigcap_{t=0}^{\infty} \tilde{Pr}^{t}(\mathcal{A})\,.$$

The set of Point-Rationalizable States of a game $\boldsymbol{u}$ satisfies

$$\mathbb{P}_{\mathcal{A}} \equiv \mathbb{P}'_{\mathcal{A}}$$

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So $\mathbb{P}'_{\mathcal{A}} \neq \mathbb{P}_{\mathcal{A}}$, which is in fact $\mathbb{P}_{\mathcal{A}} \equiv \{\mathbf{a}^*\}$.

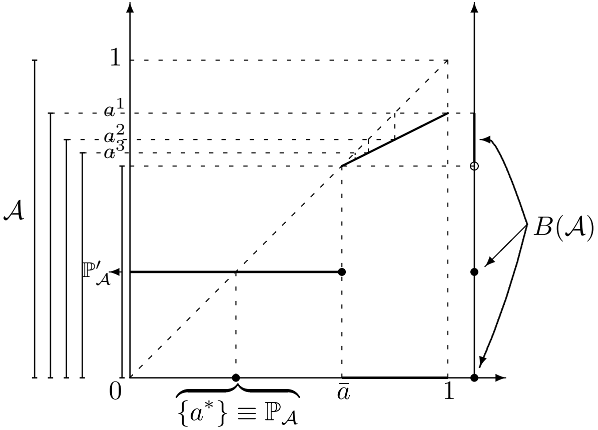

Figure: The set of Point-Rationalizable States is not the set $\mathbb{P}'_{\mathcal{A}}$.

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- Existence : $i \rightrightarrows B(i, \{a\}) \equiv \Gamma(a)$
- (Point-)Rationalizability : $i \rightrightarrows B(i, X)$

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$$\begin{array}{rll} & F^0(i) := S & \forall\, i \in I \\ \forall\, i \in I & F^t(i) := B\Big(i, \tilde{P}r^{t-1}(\mathcal{A})\Big) & t \geq 1 \end{array}$$

we have that $\tilde{P}r^{t}(\mathcal{A}) \equiv \int_I F^t(i)\ \mathrm{di}$.

- $\forall\ i \in I$ the mappings $B(i,\cdot\,) : \mathcal{A} \rightrightarrows S$ are u.s.c. and $B(i, X)$ is compact for any compact subset $X \subseteq \mathcal{A}$.

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- Define $F : I \rightrightarrows S$ as the point-wise lim sup of F^t:

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$$a \in \int_I F \mathrm{di} \subseteq \int_I B(i, \mathbb{P}'_{\mathcal{A}}) \, \mathrm{di} \equiv \tilde{Pr}(\mathbb{P}'_{\mathcal{A}}) \,.$$

$\blacksquare$

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Corollary 8

The set of Point-Rationalizable States of a game $\boldsymbol{u}$ *is well defined, non-empty, compact and convex.*

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- When we consider *standard* Rationalizability, forecasts are subjective probability distributions over the sets of outcomes.
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- In finite player games, we consider, for each player, product measures over the set of strategies of the opponents.
- In continuous player games, not trivial.
- In Rath's setting, forecasts can be assumed to be (subjective) probability distributions over the set of states.

$\mathbb{B}(i, \cdot\,) : \mathcal{P}(\mathcal{A}) \rightrightarrows S$:

$$\mathbb{B}(i, \mu) := \operatorname{argmax}_{y \in S} \mathbb{E}_{\mu}\left[u(i, y, a)\right]$$

The process of elimination of non-best-replies to (general) forecasts is described with the mapping $\tilde{R} : \mathcal{B}(\mathcal{A}) \to \mathcal{P}(\mathcal{A})$:

$$\begin{aligned}\tilde{R}(X) &:= \left\{ \int_I s(i)\,\mathrm{di} : \begin{array}{l} \mathbf{s} \in S^I,\ \mathbf{s} \text{ is a measurable selection} \\ \text{of } i \rightrightarrows \mathbb{B}(i, \mathcal{P}(X)) \end{array} \right\}. \\ &\equiv \int_I \mathbb{B}(i, \mathcal{P}(X))\,\mathrm{di}\end{aligned}$$

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Proposition 9

In a game $\boldsymbol{u}$*, if* $X \subseteq \mathcal{A}$ *is nonempty and closed, then* $\tilde{R}(X)$ *is nonempty, convex and closed.*

The Eductive Procedure: on each iteration, the states that are not reached by the process $\tilde{R}$ are eliminated:

$$\tilde{R}^0(\mathcal{A}) := \mathcal{A}, \qquad \tilde{R}^{t+1}(\mathcal{A}) := \tilde{R}\Big(\tilde{R}^t(\mathcal{A})\Big)\,.$$

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Theorem 10

In a game $\boldsymbol{u}$, the set $\mathbb{R}'_{\mathcal{A}}$ is non empty, convex and closed.

Definition 11

The set of Rationalizable States is the maximal subset $X \subseteq \mathcal{A}$ that satisfies:

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The proof mimics that of Theorem 6, taking into account that if X is compact, then when $\mathcal{P}(X)$ is endowed with the weak* topology, we preserve continuity properties of payoffs and $\mathcal{P}(X)$ is compact and metrizable, (since we use the norm in $\mathbb{R}^n$).

Proposition 13

If in a game $\boldsymbol{u}$*, we have* $\forall\ \mu \in \mathcal{P}(\mathcal{A})$*:*

$$\mathbb{E}_\mu\left[u(i,y,a)\right] \equiv u(i,y,\mathbb{E}_\mu\left[a\right])$$

then

$$\mathbb{P}_{\mathcal{A}} \equiv \mathbb{R}_{\mathcal{A}}$$

Proposition 13 says that if the utility functions are affine in the state variable, then we have that the Point-Rationalizable States set is equal to the set of Rationalizable States.

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Iterative elimination of unreasonable prices

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$$p \in P\left(\int_I \text{Supply}(i)([0, p_{max}]) \,\mathrm{di} \right)$$

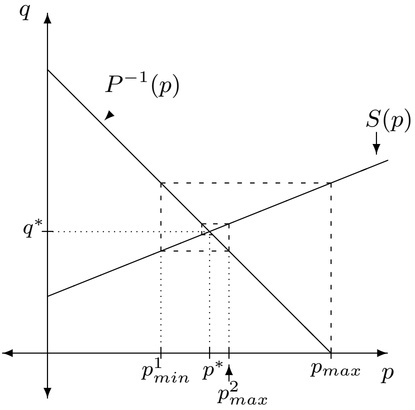

Figure: The *eductive process*

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For more results see:

- Pedro Jara-Moroni.
  Rationalizability in games with a continuum of players.
  Paris School of Economics WP, 2007.
- Roger Guesnerie and Pedro Jara-Moroni.
  Expectational coordination in a class of economic models : strategic substitutabilities versus strategic complementarities.
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Finite action set

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1. The set of Point-Rationalizable Pure Strategies $\mathbb{P}_{S_p}$
2. The set of Point-Rationalizable Mixed Strategies $\mathbb{P}_{S_m}$
3. The set of Rationalizable Pure Strategies $\mathbb{R}_{S_p}$
4. The set of Rationalizable Mixed Strategies $\mathbb{R}_{S_m}$
5. The set of Point-Rationalizable States $\mathbb{P}_{\mathcal{A}}$
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- which in turn is paired with the set of Point-Rationalizable Mixed Strategies.

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- We have defined the set of Rationalizable States and we have characterized it as the result of a process of elimination of non-best-replies to probability forecast profiles.
- This gives a general framework in which Eductive Stability may be studied (for instance Guesnerie and Jara-Moroni (2007)).

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